

Matrix at strong coupling

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Abstract

We describe the strong coupling limit ($g \rightarrow \infty$) for the Yang–Mills type matrix models. In this limit the dynamics of the model is reduced to one of the diagonal components which is characterized by a linearly confining potential. We also shortly discuss the case of the pure Yang–Mills model in more than one dimension.

1 Introduction

The development of string and gauge theories is characterized by their strong inter-relations. The most intriguing result of this interaction is, probably, the AdS/CFT conjecture [1, 2] (see [3] for a classical review of the subject). This conjecture relates the string theory in the Anti-de Sitter background on the one side with the conformal theory on the Minkowski space-time on the other. The Minkowski space-time of the conformal theory in this case is related to the (conformal) boundary of the Anti-de Sitter space.

This conjecture relates a weak coupled model to a strong coupled one and vice-versa, which is a true Ising-type duality. Once proved, it would have an immense predictive force, e.g. for describing the strong coupled dynamics of both strings and gauge fields. On the other hand, it is clear, that for a direct proof, one needs to know the strong coupled behavior of at least one of these models (in addition to the weak coupled one for the both). A considerable

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progress was achieved in recent years on the way of indirect proofs of the correspondence (for a recent review see e.g. [4]).

On the other hand, in spite of difficulties in the description, it seems, that the strong coupled regime of the gauge models is the most natural regime realized in the Nature at the most common (i.e. low) energies. Perhaps, the most success in the description of the strong coupled gauge theories was achieved in the framework of the lattice formulation.¹ An important problem of this approach, however, is that the continuum limit of strong coupled systems is problematic and it is difficult to separate the real physical effects of the strong coupling from the artifacts of the lattice description. Therefore, it would be important to have a strong coupling approach not related to the lattice discretization. In the present work we attempt to move into this direction.

Although at the end we also consider the Yang–Mills model, the main subject of this paper is the BFSS type matrix model alias Yang–Mills mechanics. Yang–Mills type matrix models appear in both contexts of string and gauge theory. Thus, BFSS [6] and IKKT [7] matrix models were proposed to describe, respectively the “zero”- and “minus-one”-brane configurations in the nonperturbative string approach (M-theory). They can be obtained as dimensional reductions to, respectively, 1+0 and 0 dimensions of the ten dimensional super Yang–Mills model. (See e.g. [8] for a review.)

The plan of the paper is as follows. In the next section we shortly introduce the matrix model. Then we consider the $g \rightarrow \infty$ limit of the matrix model. First, as a warmup we consider what we call a strong limit. In this limit we do not consider the contribution from the high frequency modes. It leads to a model for the diagonal components where all fields are statistically confined (condensed) to a single value. Next, we consider a more refined weak limit where we take into consideration the above higher modes. This leads to a dynamically nontrivial model for the diagonal components which are interacting by linear attracting potential. In addition this model appears to be semi-classical as g goes to infinity. At the end of this section we discuss a possibility for a systematic expansion at large coupling.

At the end we discuss the possibility to extend the analysis to the Yang–Mills model.

¹A good reference for the lattice approach to gauge theories is given by [5].

2 Matrix model

Consider the matrix model (Yang–Mills mechanics) which is described by the following classical action:

$$S = \int dt \text{tr} \left\{ \frac{1}{2}(\nabla_0 X_a)^2 + \frac{g^2}{4}[X_a, X_b]^2 \right\}, \quad (2.1)$$

where X_a are D , $a = 1, \dots, D$ time dependent Hermitian $N \times N$ matrices while g is the gauge coupling. The covariant time derivative is defined by the use of the (non-dynamical) temporal gauge field $A \equiv A_0$,

$$\nabla_0 X_a = \dot{X}_a + [A, X_a]. \quad (2.2)$$

The role of the gauge field is to impose the Gauss law constraint $[X_a, \nabla_0 X_a] = 0$ which provides the gauge invariance of the action with respect to the time-dependent $U(N)$ gauge transformations,

$$X_a \mapsto U^{-1}(t)X_aU(t), \quad A \mapsto U^{-1}(t)AU(t) + U^{-1}\dot{U} \quad (2.3)$$

where $U(t) \in U(N)$. Other features of the model include:

- Invariance with respect to shifts by a constant scalar matrix

$$X_a \mapsto X_a + c_a \cdot \mathbb{I}. \quad (2.4)$$

Restricting the gauge group to $SU(N)$ removes this degree of freedom

- Invariance with respect to the (target space) rotations,

$$X_a \mapsto \Lambda_a^b X_b, \quad (2.5)$$

$$\Lambda \in SO(D)$$

- In the case of $D = 10$ eq. (2.1) represents the bosonic part of the supersymmetric BFSS matrix model [6],

$$\int dt \text{tr} \left\{ \frac{1}{2}(\nabla_0 X_a)^2 + \frac{g^2}{4}[X_a, X_b]^2 + \psi \nabla_0 \psi + \psi \Gamma^a [X_a, \psi] \right\}, \quad (2.6)$$

where ψ is the fermionic $N \times N$ matrix with 10 dimensional Majorana–Weyl fermionic indices.

For the matrix model under consideration one can formulate a perturbative expansion in terms of the powers of the gauge coupling g similar to the perturbative expansion of the Yang–Mills theory. In what follows we will not discuss this type of perturbative expansion but refer the reader to the appropriate Yang-Mills perturbation theory literature instead.

3 Spontaneous symmetry breaking at strong coupling

It is expected that the strong coupling limit $g \rightarrow \infty$ implies the commutativity of the matrices X_a ,

$$[X_a, X_b] = 0. \quad (3.1)$$

Indeed, as g goes to infinity the path integral contribution of configurations with non-zero commutator are exponentially suppressed.

Since this is the case, one can diagonalize simultaneously all the matrices X_a , whose eigenvalues would then correspond to the coordinates of some branes. In this case one can say that in the strong coupling limit the branes can be localized. (Beyond this limit they are fuzzed by the strings by which the branes interact.)

Let us consider the above $g \rightarrow \infty$ limit in more details. For this let us split the matrix degrees of freedom X_a^{mn} into the diagonal part:

$$x_a = \text{diag } X_a, \quad (3.2)$$

and the remaining off-diagonal one:

$$z_a = X_a - x_a. \quad (3.3)$$

This splitting seems somehow abusive, since it does not respect the gauge invariance (2.3). In fact, it corresponds to a particular choice of commutative background among gauge equivalent ones. This choice breaks *spontaneously* the $U(N)$ symmetry down to $U(1)^N$. At the same time z_a can be treated as a perturbation above this background.

The spontaneous breaking of the symmetry is always associated with the zero modes corresponding to different gauge equivalent choices of the background². An appropriate choice of $SU(N)$ gauge apparently solves this problem since it restricts the allowed perturbations of the vacuum to the transversal direction. The unbroken gauge symmetry as well as the possibility to fix the gauge depends strongly on whether the diagonal background is degenerate or no. Although the exceptional configurations with the degenerate background may in principle contribute (and even dominate) in spite of zero measure, we so far neglect this issue and consider in rest of this paper the general position point: where all diagonal eigenvalues x_n are different (as D -dimensional vectors).

²In the present case this is the symmetry: $x_a \rightarrow U^{-1}x_aU$.

4 $g \rightarrow \infty$: the strong limit

One can define different strong coupling limits depending on the relation of the coupling with other parameters (like N or the cut-off). In this section we consider the *strong limit*: This limit assumes that the model is UV-regularized and the limit $g \rightarrow \infty$ is taken prior to removing the regularization. Technically, this means that one can drop in this limit the time derivatives if they come with a factor vanishing in the limit $g \rightarrow \infty$. In contrast to this, the *weak limit* which is taken after the removal of (or eventually not imposing at all) the regularization is discussed in the next section.

In the non-degenerate case the whole $U(N)$ gauge group is broken by the diagonal component of the background down to $U(1)^N$. The infinitesimal gauge transformation of the background is given by $\delta x_a = 0$ and $\delta z_a = [x_a, u] + [z_a, u]$. This is very similar to the ordinary gauge transformation in the nonabelian Yang–Mills theory if the role of the partial derivative operator ∂_a is attributed to the commutator $[x_a, \cdot]$. In the complete analogy with this one can fix the gauge by imposing the Lorenz gauge condition³:

$$F_{\text{g.f.}} \equiv [x_a, z_a] = 0. \quad (4.1)$$

The Faddeev–Popov determinant corresponding to the gauge fixing condition (4.1) is given by

$$\Delta_2^{(\infty)}(x) = \prod_{\text{time}} \left[\prod'_{mn} (x_a^m - x_a^n)^2 \right]^{\frac{1}{2}}, \quad (4.2)$$

where the prime denote that the product extends over the distinct indices m and n only. Formally, the determinant is different from zero (which is important for the implementation of the gauge condition) when all x -eigenvalues are given by distinct points x_a^n , $n = 1, \dots, N$.

All above can be appropriately formalized in the quantum theory by adding the gauge fixing term and the Faddeev–Popov determinant in the (Euclidean) partition function which takes the form

$$Z = \int [dx][dz][dA] \Delta_2^{(\infty)}(x) \exp\left\{-(S + \frac{g^2}{2} \text{tr}[x_a, z_a]^2)\right\}, \quad (4.3)$$

where we used so called “alpha-gauge” (with $\alpha = g^2$) implementation of the gauge fixing rather than the “delta-function implementation”. Note, that

³Admissible gauge fixing and corresponding Faddeev–Popov determinants are discussed in the classical book on gauge theories [9].

since the introduction of the gauge fixing condition (4.1) one cannot anymore impose any further restriction⁴ on the gauge field A which should remain in the action.

Now we are ready to take the limit $g \rightarrow \infty$ and separate the leading contribution in this limit. There are several ways to do this, which, naturally, lead to the same result. Let us consider the following one. Let us substitute the variables z_a by the rescaled ones as follows

$$z_a \rightarrow g z_a. \quad (4.4)$$

Then, the matrix action (2.1) takes the following form:

$$S = - \int dt \left(\frac{1}{2} \dot{x}_a^2 + \frac{1}{2g^2} \dot{z}_a^2 + \frac{1}{g} [A, z_a] \dot{x}_a + \frac{1}{2} [A, (x_a + \frac{1}{g} z_a)]^2 + \frac{1}{g} [A, (x_a + \frac{1}{g} z_a)] \dot{z}_a + \frac{1}{4} ([x_a, z_b] - [x_b, z_a] + \frac{1}{g^2} [z_a, z_b])^2 + \frac{1}{2} [x_a, z_a]^2 \right). \quad (4.5)$$

As we are taking the strong $g \rightarrow \infty$ limit, we should discard all terms formally vanishing in this limit. Thus, the leading part of the action becomes

$$S_{g \rightarrow \infty} = - \int dt \left(\frac{1}{2} \dot{x}_a^2 + \frac{1}{2} [A, x_a]^2 + \frac{1}{2} [x_a, z_b]^2 \right). \quad (4.6)$$

The action is quadratic in the gauge field A as well as in the off-diagonal field z_a . Integrating in both A and z_a , one gets the factor coinciding with the Faddeev–Popov determinant at the power $-(D+1)/2$. The partition function then reads,

$$Z = \int \left[dx \Delta_2^{(\infty)}(x)^{-(D-1)/2} \right] \exp \left\{ \int dt \frac{1}{2} \dot{x}^2 \right\}, \quad (4.7)$$

which apart from the determinant factor in the measure corresponds to a free particle partition function.

The modification of the measure in (4.7) signals the confining of the eigenvalues x_n to a common value which itself is a subject to free motion. Indeed, in the case of only two eigenvalues the path integral (4.7) reduces to (see the Appendix A),

$$Z = \int [d^D Y] [d^D y y^{-2(D-1)}] e^{i \int dt (\frac{1}{2} \dot{Y}^2 + \frac{1}{2} \dot{y}^2)}, \quad (4.8)$$

where y is the distance between branes while Y is the free moving “center of mass”. Consider the y -measure locally at the instant t : $d^D y (y^2)^{-D+1}(t) =$

⁴Except for the vanishing of the diagonal part of A , $A_{nn} = 0$.

$d\Omega_D dr r^{-D+1}$. Integration with such a measure is divergent at $r \equiv \sqrt{y^2} = 0$ unless the integrand vanishes quickly enough as y approaches the origin, which is not the case for slow y modes. Statistically this means that configurations with small y^2 produce a contribution to the partition function which is infinitely larger than the contribution of all the configurations with larger values of y^2 . Therefore, under the normalization the configurations with *nonzero* y^2 will get *zero* expectation values. One can see also that the conclusion is very sensitive to the power of $\Delta_2^{(\infty)}$. Thus, if the power were e.g. $-(D-1)/4$ no such statistical confinement would occur.

It may appear however that this simple estimation of $g \rightarrow \infty$ is too rough and one must weaken the limit allowing the contribution of higher frequency modes. We come to this in the next section.

5 $g \rightarrow \infty$: the weak limit

Consider the stationary points of the action (2.1) i.e. the solutions to the equations of motion. There is a class of static solutions to the equations of motion given by constant commuting matrices x_a . We can assume that these matrices depend adiabatically on time. One can consider perturbations about this background. The perturbation is given by the off-diagonal part z_a as well as by the fast diagonal modes. The diagonal modes do not contribute at the one-loop level since there are no nonlinear terms in the action corresponding to diagonal-diagonal interaction. As a sequence, we can neglect the fast diagonal fluctuations and consider only the adiabatic modes.

Therefore, consider the contribution of the off-diagonal modes as well as of the auxiliary (gauge and ghost) fields and evaluate their contribution in the one-loop approximation in $1/g$ expansion. Throughout this section we use the Euclideanized version of the theory.

To proceed with the evaluation let us fix the gauge by adding the following gauge fixing term to the Lagrangian:

$$L_{g.f.} = -\text{tr} \left(\frac{1}{2g^2} \dot{A}^2 + \frac{1}{2} [x_a, z_a]^2 \right). \quad (5.1)$$

The variation of the gauge fixing condition gives the Faddeev–Popov operator,

$$M_{FP} u = \partial_0 \nabla_0 u + [x_a, [(x_a + z_a), u]], \quad (5.2)$$

whose determinant $\Delta_{FP} = \det M_{FP}$ is the *Faddeev–Popov determinant* which we have to use together with the condition (5.1). In the one loop approximation no contribution will come from A - and z -dependent terms in the Faddeev–Popov operator. Therefore, in what follows we will discard these

terms. As a result, the Faddeev–Popov determinant restricted to one loop relevant terms takes the following form

$$\Delta_{FP}|_{(1 \text{ loop})} = \prod'_{m,n} \det \left[-\frac{1}{g^2} \partial_t^2 + r_{mn}^2 \right] \equiv \Delta_2(x), \quad (5.3)$$

where $r_{mn}^2 = (x_m^a - x_n^a)^2$ is the square distance between n -th and m -th branes and the prime denotes that the product is taken for distinct m and n .

Let us turn to the action (2.1). The matrix model action can be rewritten in the form as follows,

$$S = - \int dt \left(\frac{1}{2} (\dot{x}_n^a)^2 + \frac{1}{2g^2} |\dot{z}_{mn}^a|^2 + \frac{1}{2g^2} |\dot{A}_{mn}|^2 + \frac{1}{g^2} \dot{\bar{c}}_{mn} \dot{c}_{mn} \right. \\ \left. + \frac{1}{2} r_{mn}^2 (|z_{mn}^a|^2 + |A_{mn}|^2 + \bar{c}_{mn} c_{mn}) + \dots \right) \quad (5.4)$$

where the dots stand for the terms not contributing at the one loop level (e.g. terms which are higher than the second order in A and z).

After the integration over the gauge field A , the off-diagonal component z and the ghosts c and \bar{c} the partition function takes the form,

$$Z = \int [dx] \Delta_2^{-\frac{D-1}{2}}(x) e^{\int dt \frac{1}{2} \dot{x}^2} \quad (5.5)$$

As it can be seen, the problem is reduced to the computation of the determinant Δ_2 , of an elliptic differential operator. Let us use the ζ -function approach to do such a computation⁵ (see e.g. [10]). According to this approach, the logarithm of the determinant of an elliptic operator D is given by the (minus) derivative of the ζ -function,

$$\ln \det D = -\zeta'_D(0), \quad (5.6)$$

where the function $\zeta_D(s)$ is defined as the analytic continuation of the series,

$$\zeta_D(s) = \sum_{\lambda} \frac{1}{\lambda^s} = \frac{1}{\Gamma(s)} \int_0^{\infty} d\rho \rho^{s-1} \text{tr } e^{-\rho D}. \quad (5.7)$$

The trace $\text{tr } e^{-\rho D}$ can be written as

$$\text{tr } e^{-\rho D} = \int dt K_D(t, t; \rho), \quad (5.8)$$

⁵A similar computation for constant diagonal modes was done in [14] and its phenomenological implications were explored in [15]. I thank Amir H. Fathollahi for pointing my attention to these papers.

where $K_D(t', t''; \rho)$ is the Heat Kernel for the operator D . It is the solution to the Heat Equation

$$\partial_\rho K(t, t_0; \rho) = -DK(t, t_0; \rho), \quad (5.9)$$

with the initial conditions given by

$$K(t, t_0; 0) = \delta(t - t_0). \quad (5.10)$$

In the case at hand $D = \left(-\frac{1}{g^2} \partial_t^2 + r_{mn}^2\right)$ and the solution for the Heat Kernel is given by

$$K(t', t''; \rho) = \frac{g}{\sqrt{4\pi\rho}} \exp\left(-\frac{g^2(t'' - t')^2}{4\rho} - r_{mn}^2\rho\right). \quad (5.11)$$

Since the time integral in the r.h.s of the equation (5.8) diverges for $t \in (-\infty, +\infty)$ it is useful to put the system in the time box interval τ . Beyond its regularization function the τ plays another important role, namely, that of being also the *adiabaticity* box. Roughly speaking, the τ -interval is the “dt” for the adiabatic time “t”.

The ζ -function for the time interval τ is then given by

$$\begin{aligned} \zeta_D(s) &= \frac{g\tau}{\sqrt{4\pi}\Gamma(s)} \sum'_{mn} \int_0^{+\infty} d\rho \rho^{s-3/2} e^{-r_{mn}^2\rho} \\ &= \frac{g\Gamma(s-1/2)}{\sqrt{4\pi}\Gamma(s)} \tau \sum'_{mn} (r_{mn}^2)^{1/2-s}. \end{aligned} \quad (5.12)$$

Computing the derivative of (5.12) and taking the limit $s \rightarrow 0$ we obtain:

$$-\zeta'_D(0) = g\tau \sum'_{mn} \sqrt{r_{mn}^2}. \quad (5.13)$$

Summing over the all adiabatic boxes we get:

$$\Delta_2^{-\frac{D-1}{2}}(x) = e^{-\frac{g(D-1)}{2} \int dt \sum_{mn} \sqrt{r_{mn}^2}}, \quad (5.14)$$

where we can even drop the prime from the sum.

Therefore, the low energy effective action for x_n^a takes the form,

$$S_{g \rightarrow \infty} = \int dt \left(-\frac{1}{2} \dot{x}_n^2 - \frac{1}{2} g(D-1) \sum_{mn} \sqrt{(x_m - x_n)^2} \right). \quad (5.15)$$

As one can see, the action (5.15) corresponds to a system with strong linear confinement of the particles. In spite of its terrifying appearance the limit $g \rightarrow \infty$ corresponds to nothing else than the semi-classical limit. Indeed, passing to a rescaled x_n^a ,

$$x_n^a \rightarrow x_n^a/g(D-1), \quad (5.16)$$

transforms the partition function (5.15) to the following semi-classical form

$$Z = \int [dx] e^{g^2(D-1)^2 \int dt \left(-\frac{1}{2} \dot{x}_n^2 - \frac{1}{2} \sum_{mn} \sqrt{(x_m - x_n)^2} \right)}, \quad (5.17)$$

where g^2 plays the role of inverse Planck constant \hbar^{-1} . In fact, the above rescaling introduces a renormalization of the brane coordinate. Its meaning is that the nontrivial dynamics corresponds to large (in the old scale) brane separations. Therefore, the natural scale of the brane dynamics is given in terms of the attraction force (tension) acting on the branes.

A remark on the systematic expansion

A trick can be used to modify the value of the coupling constant (and even to invert it).

We can consider the model at the finite temperature $T = 1/\beta$. The finite temperature implies that the action in the path integral is computed for the Euclidean time interval $0 \leq t < \beta$ with periodical boundary condition for the fields. A simple dimensional analysis shows that the following rescaling

$$\beta \rightarrow \beta/\lambda^2, \quad g^2 \rightarrow g^2 \lambda^6, \quad (5.18)$$

changes the partition function by a constant multiplicative factor only, which can be absorbed in the measure. Indeed, making the substitution $X \rightarrow \lambda X$ one gets (5.18). Now taking λ arbitrarily small one can make g small as well, e.g. equal to g^{-1} . At the same time β goes to infinity i.e. the theory rolls down to zero temperature.

Unfortunately, because of different scaling properties, this trick can not be used in the case of Yang–Mills theory in more than two dimensions.

6 The Yang–Mills model

It is tempting to apply the above analysis to the $SU(N)$ Yang–Mills model. Let us enumerate the modifications that occur when passing to the pure D -dimensional Yang–Mills model:

- Instead of the determinant (5.3) one should compute the determinant of the D -dimensional differential operator

$$D = \frac{1}{g^2} \partial_\mu^2 - r_{mn}^2, \quad (6.1)$$

where the diagonal modes are described by the Abelian gauge fields $a_\mu^n(x)$, $r_{mn}^2 = (a_\mu^m - a_\mu^n)^2$. Also since the gauge group is $SU(N)$ the center of mass is fixed:

$$\sum_n a_\mu^n = 0. \quad (6.2)$$

- Heat Kernel:

$$K(x', x''; \rho) = \frac{g^D}{(4\pi\rho)^{D/2}} \exp\left(-\frac{g^2(x'' - x')^2}{4\rho} - r_{mn}^2\rho\right). \quad (6.3)$$

- The one loop contribution is given by:⁶

$$L_{eff} = \frac{1}{4}(F_{\mu\nu}^n)^2 - g^D(D-2) \sum_{mn} V_{mn}(a), \quad (6.4)$$

where $F_{\mu\nu}^n = \partial_\mu a_\nu^n - \partial_\nu a_\mu^n$ and

$$V_{mn} = \frac{(r_{mn}^2)^{D/2}}{(4\pi)^{D/2}} \times \begin{cases} \Gamma(-D/2), & D\text{-odd} \\ \frac{(-1)^{D/2}}{(D/2)!} (\log r_{mn}^2 - h(D/2)), & D\text{-even} \end{cases} \quad (6.5)$$

where $h(k)$ is the k -th harmonic number: $h(k) = \sum_{l=1}^k 1/l$. (Note also that the Γ -function is regular at negative half-integer points.)

As it could be seen, for $D > 2$ ($D = 2$ is dynamically trivial) one can rescale the fields

$$a_\mu^n \rightarrow g^{\frac{D}{D-2}} a_\mu^n, \quad (6.6)$$

and get a common factor $g^{\frac{2D}{2-D}}$ in front of the effective action. For $D > 2$ this factor vanishes in the limit $g \rightarrow \infty$, which means that in this case the quantum fluctuations are strong. As one can note, the the qualitative behaviour of the effective models depends on dimension. Thus, in dimensions $D = 4k$ and $4k + 1$ for non-negative integer k , the strong attractive force binds all a^n together, while for $D = 4k + 2$ and $4k + 3$ the repulsive force keeps them apart at infinity. The common feature is that in this situation

⁶The factor $(D-2)$ instead of $(D-1)$ as in the previous sections appears because the gauge field A_0 is now counted as one of the fields.

we are not able to catch any nontrivial dynamics beyond the fact that all diagonal values are confined to zero or infinity.

A much more serious problem is that for $D > 2$ the higher loop contribution is not suppressed at large g unless an UV-cutoff ($\Lambda < \infty$) is used. A nontrivial contribution can be then caught taking the double scaling limit with $g \rightarrow \infty$ and $\Lambda \rightarrow \infty$. A more detailed analysis would give the answer whether this is possible.

7 Discussion

In this paper we considered the strong coupling limit of the matrix model. It is shown that the modes which survive in this limit are described by a system of linearly interacting particles. As coupling goes to infinity the system becomes semi-classical g^2 playing the role of inverse the inverse Planck constant \hbar^{-1} . The scale at which the dynamics takes the semi-classical form is given then by the string tension or the coefficient of linear interaction. The analysis is performed at the one-loop level. It seems rather possible that a systematic expansion in the inverse powers of the coupling constant can be constructed in addition to the standard small coupling constant expansion.

It is also very tempting to apply the $1/g$ -expansion to the Yang–Mills theory. The one-loop technique can be easily extended to the ordinary Yang–Mills model. In the case of the dimensionality higher than two the diagonal component is not anymore semi-classical and most probably does not decouple. The implications of this are not yet clear. There are however, resources we did not use which are given by the large N and UV cut-off scaling. Taking a correlated limit of large g , N and Λ one may hope to get a non-trivial content for the expansion, e.g. by tuning the background.

Another important issue we left beyond our consideration regards the exceptional configurations with some $r_{mn} = 0$. As the effective parameter of the expansion is $1/gr_{mn}$ the expansion fails if some $r_{mn} \lesssim g^{-1}$. Important point is the statistical weights of such configurations. An estimate can be done by the computation of the average separation \bar{r} . When the average separation is nonzero $\bar{r} > 0$, it is clear, that one can trust the approach. In the case of pure Yang–Mills model, however, it seems that it either vanishes or is infinite according to the dimension.

In the case of branes at close distance the 2×2 matrix block corresponding to respective eigenvalues is not decoupled and one should consider the entire matrix dynamics similarly to what is done in the non-commutative gauge theory [11, 12, 13]. As it was found this dynamics is a stochastic one.

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A Example: The tale of two branes

Consider the case of two branes. In this case the action can be written in the following form:

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}\dot{Y}^2 + \frac{1}{2}\dot{y}^2 + \dot{z} \cdot \dot{\bar{z}} \\ & + \sqrt{2}[a(\dot{y} \cdot \bar{z} - \dot{\bar{z}} \cdot y) - \bar{a}(\dot{y} \cdot z - \dot{z} \cdot y)] - (a_1 - a_2)(\dot{z} \cdot \bar{z} - \dot{\bar{z}} \cdot z) \\ & + \sqrt{2}(a_1 - a_2)(ay \cdot \bar{z} + y \cdot z\bar{a}) - 2a\bar{a}(z \cdot \bar{z} + y^2) + z^2\bar{a}^2 + a^2\bar{z}^2 \\ & - \frac{1}{2}z \cdot \bar{z}(a_1 - a_2)^2 \\ & - g^2(2y^2z \cdot \bar{z} + (z \cdot \bar{z})^2 - z^2\bar{z}^2) - 2y^2(c\bar{c} + c^*\bar{c}^*), \end{aligned} \quad (\text{A.1})$$

where 2×2 matrix X_a is given by the following component structure,

$$X_a = \begin{pmatrix} \frac{1}{\sqrt{2}}(Y_a + y_a) & z_a \\ \bar{z}_a & \frac{1}{\sqrt{2}}(Y_a - y_a) \end{pmatrix}, \quad (\text{A.2})$$

while the gauge field matrix A is given by the components:

$$A = \begin{pmatrix} a_1 & a \\ \bar{a} & a_2 \end{pmatrix}, \quad (\text{A.3})$$

and two complex conjugate components of the ghost-anti-ghost are used. All diagonal components are real while the off diagonal elements are complex. The dot in (A.1) indicates the inner product with respect to the index $a = 1, \dots, D$.

The first four lines of (A.1) are the contribution from the kinetic term while the third line comes from the commutator term together with the gauge fixing term and Faddeev–Popov determinant for the gauge $[x, z] = 0$.

Let us make the following substitution:

$$z \rightarrow gz. \quad (\text{A.4})$$

After the rescaling one can split the Lagrangian (A.1) in the leading term and perturbation in $1/g$. The leading part of the Lagrangian looks as follows,

$$\mathcal{L}_{g \rightarrow \infty} = \frac{1}{2}\dot{Y}^2 + \frac{1}{2}\dot{y}^2 - 2y^2(a\bar{a} + z \cdot \bar{z} + c\bar{c} + c^*\bar{c}^*). \quad (\text{A.5})$$

All fields with the exception of the y and the free Y become non-dynamical in the limit $g \rightarrow \infty$ and the Lagrangian (A.5) is quadratic this fields. Therefore, integration of z , \bar{z} , c , \bar{c} and a leads⁷ to the partition function of the form (4.7),

$$Z = \int \prod_t d^D Y d^D y [y^2(t)]^{-D+1} e^{i \int dt \left(\frac{1}{2} \dot{Y}^2 + \frac{1}{2} \dot{y}^2 \right)}. \quad (\text{A.6})$$

As in the case of (4.7) the measure in eq. (A.6) is singular as $y^2 \rightarrow 0$.

References

- [1] J.M. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231, hep-th/9711200.
- [2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys. Lett. B428 (1998) 105, hep-th/9802109.
- [3] O. Aharony et al., Phys. Rept. 323 (2000) 183, hep-th/9905111.
- [4] J. Plefka, (2005), hep-th/0507136.
- [5] M. Creutz, Quarks, gluons, and lattices (Cambridge University Press, 1985).
- [6] T. Banks et al., Phys. Rev. D55 (1997) 5112, hep-th/9610043.
- [7] N. Ishibashi et al., Nucl. Phys. B498 (1997) 467, hep-th/9612115.
- [8] C. Sochichi, (2005), hep-th/0506186.
- [9] L. Faddeev and A. Slavnov, Gauge fields, introduction to quantum theory (Addison-Wesley Pub., 1991).
- [10] A.S. Schwarz, Quantum field theory and topology (Springer-Verlag, 1993).
- [11] C. Sochichi, J. Phys. A35 (2002) 3125, hep-th/0104076.
- [12] C. Sochichi, Czech. J. Phys. 51 (2001) 1447, hep-th/0109157.
- [13] C. Sochichi, (2001), hep-th/0109158.

⁷Remaining components contribute by only a constant factor.

- [14] A. H. Fatollahi, *Europhys. Lett.* **53** (2001) 317 [arXiv:hep-ph/9902414];
A. H. Fatollahi, *Eur. Phys. J. C* **19** (2001) 749 [arXiv:hep-th/0002021].
- [15] A. H. Fatollahi, *Phys. Rev. D* **65** (2002) 046004 [arXiv:hep-th/0108198];
A. H. Fatollahi, *Eur. Phys. J. C* **21** (2001) 717 [arXiv:hep-th/0104210];
A. H. Fatollahi, *Eur. Phys. J. C* **27** (19??) 145 [arXiv:hep-ph/0203115];
A. H. Fatollahi, *Mod. Phys. Lett. A* **19** (2004) 1577
[arXiv:hep-ph/0306150].